Enhanced method for single-antenna GPS-based attitude determination

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Abstract

Purpose – To make the single-antenna attitude method more useful as a back-up or fault diagnostic system than was targeted originally.

Design/methodology/approach – The enhancement incorporates information from the GPS satellite constellation and aircraft dynamic model. The visibility of GPS satellites affects the accuracy of the aircraft's volocity that is the main source of single-antenna attitude. In addition, to use the aircraft dynamic model is natural because single-antenna attitude is for exclusive use of aircraft. These are considered and implemented as a covariance matrix or process model of Kalman filters. The enhanced performances are verified by an aircraft nonlinear simulation.

Findings – The proposed method estimates more accurate volocity and unpiased single-antenna attitude by using satellite constellation information and the aircraft dynamics. Moreover, the implemented system has a structure that combines other navigation sensors easily.

Research limitations/implications – It would be more desirable to perform further researches; sensor integration, stability against wind disturbance, and aircraft model uncertainty, etc.

Practical implications – A useful attitude sensor for a back-up attitude system at low cost on manned aircraft or a main attitude system on unmanned aircraft that are sensitive to the mass or size of payload.

Originality/value – This paper has been the first to promote the potential of single-antenna attitude and with only information that can be easily obtained.

Keywords Attitudes, Aerodynamics, Velocity measurement, Non-linear control systems

Paper type Research paper

Nomenclature

$\mathbf{a}_g$ = aircraft acceleration in north, east, down (NED) axis
$\mathbf{a}_g^n$, $\mathbf{a}_g^t$ = components of $\mathbf{a}_g$ normal, tangential to $\mathbf{v}_g$
B = receiver clock bias
b = satellite clock bias
$\hat{\mathbf{e}}$ = direction vector from user antenna to satellite
$\mathbf{g}$, g = gravitational acceleration vector, magnitude
$\mathbf{g}^n$, g^t = components of $\mathbf{g}$ normal, tangential to $\mathbf{v}_g$
I_x, I_y, I_z, I_{xz} = moment of inertia, product of inertia of aircraft
$\mathbf{l}$ = lift acceleration based on $\mathbf{v}_g$
L, M, N = components of total external moment
m = mass of aircraft
$\mathbf{p}$ = horizontal reference vector perpendicular to $\mathbf{v}_g$
p, q, r = body axis components of the angular velocity
$\tilde{q}$, $\tilde{r}$ = pseudo-pitch rate, pseudo-yaw rate
$\mathbf{R}$ = position vector of satellite
$\mathbf{R}_u$ = position vector of user antenna
u, v, w = body axis components of velocity
V_T = speed of aircraft, $|\mathbf{v}_g|$
$\mathbf{v}_g$ = aircraft velocity vector in north, east, down (NED) axis
v_{gN}, v_{gE}, v_{gD} = NED axis components of $\mathbf{v}_g$
X, Y, Z = components of total external force except gravity
α, β, γ = angle of attack, sideslip angle, flight path angle
δe, δt, δa, δr = control input deflections of elevator, thrust, aileron, and rudder
ψ, θ, ϕ = attitude angle in body axis (yaw, pitch, roll)
$\tilde{\psi}$, $\tilde{\theta}$, $\tilde{\phi}$ = pseudo-attitude angle
ϕ = carrier phase measurement

1. Introduction

Since, the proposition of GPS as an attitude estimator by Spinney (1976), many algorithms have been devised and tested. Most of these algorithms have used multiple antennas and corresponding signal-tracking modules. Cohen (1992) showed that by processing the difference in carrier-phase measurements and resolving the integer ambiguity of the carrier phase, the system attitude can be determined successfully. However, the use of multiple antennas incurs relatively high capital cost. Kornfeld *et al.* (1998) suggested that a better way of providing a low cost device is to develop an efficient algorithm that enables attitude determination with only single antenna.

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The algorithm synthesizes aircraft attitude from single-antenna GPS velocity measurements under the assumption of coordinated flight using a simple point mass aircraft model. The synthesized attitude consists of horizontal and vertical flight-path angles and roll angle about the aircraft velocity vector axis.

However, the velocity axis is not aligned with the body axis of aircraft. The axis difference causes some bias in single-antenna attitude. This is why single-antenna attitude is often called a pseudo-attitude. The single-antenna attitude system cannot fully substitute for the main attitude system, including gyro sensors and others, but it can play an important role as a backup attitude system at low cost when the main attitude system in manned aircraft fails. The single-antenna attitude system can be deployed, as the main sensor, on unmanned aircraft that are sensitive to the mass or size of payload.

This paper describes an enhanced method that derives a pseudo-attitude closer to the true aircraft attitude using GPS satellite constellation and aircraft dynamic model information. The performance of the proposed method is evaluated by simulation.

The rest of the paper is organized as follows. Section 2 reviews existing work in single-antenna attitude determination and identify areas for improvement. Section 3 describes an improved single-antenna GPS-based attitude model. Section 4 describes the implementation and performance verification of the enhanced model. Section 5 summarizes the paper's contribution and suggests areas for future improvements.

2. Single-antenna attitude

2.1 Existing method

The discussion in this section has been primarily drawn from Kornfeld *et al.* (1998) and Kornfeld (1999).

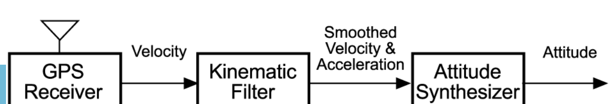
Figure 1 shows the simplified block diagram of a single-antenna GPS-based attitude-determining model. To determine a system's attitude, the model requires the velocity and acceleration of aircraft be known. Since, most of the GPS receivers provide only the position and velocity, a kinematic filter is added to smooth the velocity and estimate the acceleration. Finally, the attitude synthesizer prepares the pseudo-attitude information, which may be fed into the display console, system-controller, and other relevant sensory systems.

2.1.1 Kinematic filter

A kinematic filter is simply a Kalman filter in which the process model contains several integrators in cascade. If the system model cannot be identified precisely, this filter makes it easy to estimate and smooth the derivative state of a measured one.

The filter state-vector consists of velocity, acceleration, and jerk. The position or derivatives of jerk can be employed for the quality of estimated states. Three kinematic-filters are built-in: each filter smoothes and estimates the velocity and acceleration independently in the north, east, down (NED) direction.

Figure 1 Simplified block diagram of single-antenna attitude determination



The Kinematic-filter equations in discrete domain are expressed as follows:

$$\text{Process model}: \ \mathbf{x}_{k+1} = \Phi \mathbf{x}_k + \mathbf{w}_k \tag{1}$$

$$\text{Measurement model}: \ \mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k, \tag{2}$$

Where $(\ )_k$, at time t_k; $\mathbf{x}$, state vector, $\mathbf{x} = [velocity \ \ acceleration \ \ jerk]^T$; $\mathbf{w}$, processing noise vector; $\mathbf{v}$, measurement noise vector

$$\Phi = \begin{bmatrix} 1 & \Delta t & \Delta t^2/2 \\ 0 & 0 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

The velocity and acceleration are estimated through the propagation and innovation processes of the Kalman filter.

2.1.2 Attitude synthesizer

Pseudo-attitude is the Euler angle of the ground velocity axis because ground velocity is the basic source. Figure 2 shows the reference axes of aircraft navigation. If pseudo-attitude is determined with the relative wind vector, there may be some biases caused by the angle of attack and the sideslip angle. Moreover, the ground velocity of aircraft is generally unaligned with the wind axis because of the non-zero wind. Thus, pseudo-attitude is not equal to the true attitude until constrained conditions are fully satisfied, such as zero angle of attack, coordinated flight, and no wind disturbance.

In the existing method, pseudo-heading and pseudo-pitch angle $(\tilde{\psi}, \ \tilde{\theta})$ are defined as the horizontal and vertical flight path angle:

$$\tilde{\psi} = \tan^{-1}\left(\frac{v_{gE}}{v_{gN}}\right) \tag{3}$$

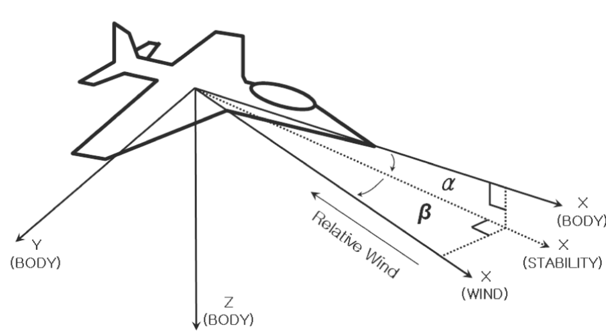
$$\tilde{\theta} = \tan^{-1}\left(\frac{-v_{gD}}{\sqrt{v_{gN}^2 + v_{gE}^2}}\right) \tag{4}$$

Pseudo-roll is defined as the effective bank angle, which corresponds to the observed lateral rate of change of the velocity vector $\mathbf{v}_g$. Ground acceleration $\mathbf{a}_g$ is separated into components that are tangential and normal to $\mathbf{v}_g$:

$$\mathbf{a}_g^t = \frac{\mathbf{a}_g \cdot \mathbf{v}_g}{|\mathbf{v}_g|^2} \cdot \mathbf{v}_g \tag{5}$$

$$\mathbf{a}_g^n = \mathbf{a}_g - \mathbf{a}_g^t \tag{6}$$

Figure 2 Definition of aircraft axes



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Similarly, the local gravitational acceleration $\mathbf{g}$ is separated into tangential and normal components:

$$\mathbf{g}^t = \frac{\mathbf{g}\cdot\mathbf{v}_g}{|\mathbf{v}_g|^2}\cdot\mathbf{v}_g \tag{7}$$

$$\mathbf{g}^n = \mathbf{g} - \mathbf{g}^t, \tag{8}$$

where $\mathbf{g} = \begin{bmatrix} 0 & 0 & g \end{bmatrix}^{\mathrm{T}}$ in NED axis.

Since, coordinated flight is assumed, lift acceleration is obtained using:

$$\mathbf{a}_g^n = \mathbf{l} + \mathbf{g}^n, \quad \text{or } \mathbf{l} = \mathbf{a}_g^n - \mathbf{g}^n, \tag{9}$$

where the horizontal reference vector is defined as (Figure 3):

$$\mathbf{p} = \mathbf{g} \times \mathbf{v}_g. \tag{10}$$

In the end, the pseudo-roll angle is obtained using:

$$\tilde{\boldsymbol{\phi}} = \sin^{-1}\left(\frac{\mathbf{l}\cdot\mathbf{p}}{|\mathbf{l}|\cdot|\mathbf{p}|}\right). \tag{11}$$

Additionally, pseudo-pitch rate and pseudo-yaw rate are determined by:

$$\tilde{q} = \frac{\mathbf{a}_g^n\cdot\mathbf{i}_L}{|\mathbf{v}_g|}, \quad \tilde{r} = \frac{\mathbf{a}_g^n\cdot\mathbf{i}_C}{|\mathbf{v}_g|}, \tag{12}$$

where:

$$\mathbf{i}_L = \frac{\mathbf{l}}{|\mathbf{l}|}, \quad \mathbf{i}_C = \frac{\mathbf{v}_g \times \mathbf{l}}{|\mathbf{v}_g \times \mathbf{l}|}.$$

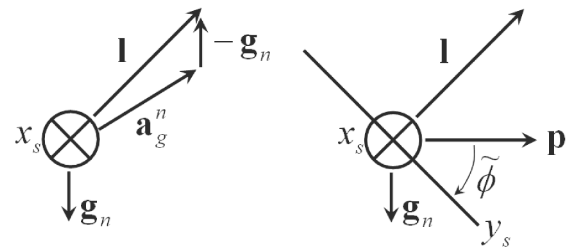
2.2 Margin of improvement

Whilst the single-antenna attitude-determining model has been shown by Kornfeld *et al.* (1998) to be cost-effective, simply realizable, and a reliable system capable of being used as an attitude backup or a fault detection system, the model requires benign atmospheric conditions and coordinated flight. In spite of this, there are areas open to improvements, including the following:

- It is well known that visibility of GPS is dependent on the aircraft attitude, which varies. In addition, the accuracy of GPS navigational sensors depends partly on the satellite constellation, which also applies to velocity. The kinematic filter of the existing method may be closer to the optimal by applying the velocity-measurement-noise covariance obtained from the satellite constellation.
- Single-antenna attitude model is for exclusive use of aircraft. Thus, the existing method included the relation of lift and gravity among the various dynamic features of aircraft. Additionally, it would be more effective to use general features (e.g. the relation between longitudinal and lateral motion) and specific features (e.g. the stability derivatives that are estimated from wind tunnel test, aerodynamic and propulsion theory, and flight tests). This modification would make the pseudo-attitude closer to the true attitude.
- The structure of attitude synthesizer combined only with kinematic filter is not adequate for processing deterministic control input or other sensor information when it is available.

The preceding discussion demonstrates the potential for enhancement of the existing model.

Figure 3 Lift acceleration and horizontal reference vector



3. Enhanced method

3.1 Satellite constellation

In GPS-aided navigation, there are several sources contributing to measurement errors, for instance, ionospheric delay, tropospheric delay, multi-path, ephemeris error, etc. As noted in Section 2, the accuracy of any navigation solution also depends on the constellation of the GPS satellites. In real life, an aircraft performs various manoeuvres, which cause the covariance of velocity error to fluctuate over the mission time.

Most GPS receivers do not report additional information such as navigation error variance/covariance values. The manufacturers of the receivers do not fully divulge the internal estimation process of their products. These problems make it difficult to optimize the kinematic filter with respect to the target's velocity.

To circumvent these problems, and given that GPS raw measurement and satellite ephemeris data are readily available (except in ultra low-priced receivers), it becomes more desirable to estimate the target's velocity outside the loop, like the velocity estimator as shown in Figure 4.

Velocity is estimated from the differential carrier phase or Doppler measurements. If the integer ambiguity and other error sources are neglected, the carrier phase observation model may be expressed as:

$$\varphi^i = (\mathbf{R}^i - \mathbf{R}_u)\cdot\hat{\mathbf{e}}^i + B - b^i + \nu_{\varphi^i}, \tag{13}$$

where $(\ )^i$, ith satellite; $v_{()}$, measurement noise.

The differential carrier phase or the Doppler observation model is obtained by differentiation. Thus:

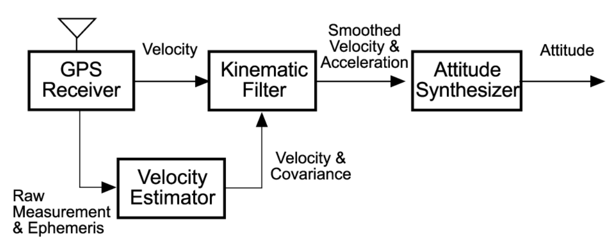
$$\dot{\varphi}^i = (\dot{\mathbf{R}}^i - \dot{\mathbf{R}}_u)\cdot\hat{\mathbf{e}}^i + \dot{B} - \dot{b}^i + \nu_{\dot{\varphi}^i}. \tag{14}$$

By rearranging (14) about the unknown, $\dot{R}_u$ and $\dot{B}$ we obtain:

$$\dot{\mathbf{R}}_u\cdot\hat{\mathbf{e}}^i - \dot{B} = \dot{\mathbf{R}}^i\cdot\hat{\mathbf{e}}^i - \dot{b}^i - \dot{\varphi}^i + \nu_{\dot{\varphi}^i}. \tag{15}$$

A matrix equation is composed after collection of the measurements for all of the n visible satellites:

Figure 4 Block diagram with velocity estimator



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$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{v}, \tag{16}$$

where $\mathbf{v}$, measurement noise vector:

$$\mathbf{x} = \begin{bmatrix} \dot{R}_u \\ \dot{B} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} \hat{e}^{1^{\mathrm{T}}} & -1 \\ \hat{e}^{2^{\mathrm{T}}} & -1 \\ \vdots & \\ \hat{e}^{n^{\mathrm{T}}} & -1 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} \dot{R}^1 \cdot \hat{e}^1 - \dot{b}^1 - \dot{\varphi}^1 \\ \dot{R}^2 \cdot \hat{e}^2 - \dot{b}^2 - \dot{\varphi}^2 \\ \vdots \\ \dot{R}^n \cdot \hat{e}^n - \dot{b}^n - \dot{\varphi}^n \end{bmatrix}.$$

The unknown vector that contains the velocity and receiver clock drift is determined by the least square estimation method. Specifically:

$$\hat{\mathbf{x}} = (\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{z}. \tag{17}$$

Covariance of estimation error $\delta\hat{\mathbf{x}}$ is:

$$\begin{aligned} Cov(\delta\hat{\mathbf{x}}) &= E(\delta\hat{\mathbf{x}}\delta\hat{\mathbf{x}}^{\mathrm{T}}) \\ &= E((\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\delta\mathbf{z}\delta\mathbf{z}^{\mathrm{T}}\mathbf{H}(\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1}) \\ &= (\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}E(\delta\mathbf{z}\delta\mathbf{z}^{\mathrm{T}})\mathbf{H}(\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1} \end{aligned} \tag{18}$$

The equation shows that covariance $Cov(\delta\hat{\mathbf{x}})$ depends not only on the covariance of raw measurement noise $E(\delta\mathbf{z}\delta\mathbf{z}^{\mathrm{T}})$ but also on the satellite constellation $\mathbf{H}$. The covariance of raw measurement noise is generally modelled as an exponential function of elevation angle of satellite. Therefore, the covariance of the estimated velocity can be determined precisely epoch by epoch. This contributes to the optimized operation of the kinematic filter.

In case of weighted least square of the diagonal matrix $\mathbf{w}$, the velocity vector and its covariance matrix may be expressed as:

$$\hat{\mathbf{x}} = (\mathbf{H}^{\mathrm{T}}\mathbf{W}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{W}\mathbf{z} \tag{19}$$

$$\begin{aligned} Cov(\delta\hat{\mathbf{x}}) &= E(\delta\hat{\mathbf{x}}\delta\hat{\mathbf{x}}^{\mathrm{T}}) \\ &= E((\mathbf{H}^{\mathrm{T}}\mathbf{W}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{W}\delta\mathbf{z}\delta\mathbf{z}^{\mathrm{T}}\mathbf{W}\mathbf{H}(\mathbf{H}^{\mathrm{T}}\mathbf{W}\mathbf{H})^{-1}) \\ &= (\mathbf{H}^{\mathrm{T}}\mathbf{W}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{W}E(\delta\mathbf{z}\delta\mathbf{z}^{\mathrm{T}})\mathbf{W}\mathbf{H}(\mathbf{H}^{\mathrm{T}}\mathbf{W}\mathbf{H})^{-1} \end{aligned} \tag{20}$$

3.2 Aircraft dynamics

Non-zero angle of attack and sideslip angle cause the pseudo-attitude estimates to differ from true attitude, even with the assumption of zero wind vector. Pseudo-roll angle is insensitive to these angles, making bias rather small. However, pseudo-pitch differs from actual pitch angle by the angle of attack if there are only the longitudinal motions. The difference may also changes if there are lateral motions in parallel.

If the single-antenna attitude model is for the exclusive use of aircraft, it is natural to apply the equations of motion of conventional aircraft. Furthermore, the dynamics information of a specific aircraft can make the single-antenna attitude model more accurate.

Section 3.2.1 summarizes the equations of motion and rate-of-climb constraint, while Section 3.2.2 discusses the proposed additional filter that utilizes the expressions in Section 3.2.1.

3.2.1 Equation of motion and rate-of-climb constraint

Equation of motion and rate-of-climb constraint can be written in several forms. We adopted following form that was written by Stevens and Lewis (1992) because it is more helpful to realize the enhancement using them.

Equation of motion:

1 Force equation:

$$\begin{aligned} \dot{u} &= rv - qw - g\sin\theta + X/m \\ \dot{v} &= -ru + pw + g\cos\theta\sin\phi + Y/m \\ \dot{w} &= qu - pv + g\cos\theta\cos\phi + Z/m. \end{aligned} \tag{21}$$

2 Momentum equation:

$$\begin{aligned} \dot{p} &= (c_1 r + c_2 p)q + c_3 L + c_4 N \\ \dot{q} &= c_5 pr - c_6(p^2 - r^2) + c_7 M \\ \dot{r} &= (c_8 p - c_2 r)q + c_4 L + c_9 N \end{aligned} \tag{22}$$

where:

$$\begin{aligned} \Gamma c_1 &= (I_y - I_z)I_z - I_{xz}^2, \\ \Gamma c_2 &= (I_x - I_y + I_z)I_{xz} \\ \Gamma c_3 &= I_z, \\ \Gamma c_4 &= I_{xz}, \\ c_5 &= \frac{I_z - I_x}{I_y}, \\ c_6 &= \frac{I_{xz}}{I_y} \\ c_7 &= \frac{1}{I_y}, \\ \Gamma c_8 &= I_x(I_x - I_y) + I_{xz}^2, \\ \Gamma c_9 &= I_x \\ \Gamma &= I_x I_z - I_{xz}^2 \end{aligned}$$

3 Kinematic equation:

$$\begin{aligned} \dot{\psi} &= q\sin\phi\sec\theta + r\cos\phi\sec\theta \\ \dot{\theta} &= q\cos\phi - r\sin\phi \\ \dot{\phi} &= p + q\tan\theta\sin\phi + r\tan\theta\cos\phi. \end{aligned} \tag{23}$$

Rate-of-climb constraint

The aircraft velocity in wind axis is

$$\begin{bmatrix} V_{\mathrm{T}} & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

and the down component $\mathbf{v}_g$ in NED axis is $v_{gD} = -V_{\mathrm{T}}\sin\gamma$ under the assumption of zero wind-vector. So, the following relation is obtained:

$$\begin{aligned} \mathbf{v}_g = \begin{bmatrix} v_{gN} \\ v_{gE} \\ v_{gD} \end{bmatrix} &= \begin{bmatrix} v_{gN} \\ v_{gE} \\ -V_{\mathrm{T}}\sin\gamma \end{bmatrix} \\ &= \mathrm{T}_{BODY\to NED}(\psi, \theta, \phi)\mathrm{T}_{WIND\to BODY}(\alpha, \beta)\begin{bmatrix} V_{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix} \end{aligned} \tag{24}$$

where:

$$\tan\alpha = \frac{w}{u}, \quad \sin\beta = \frac{v}{\sqrt{u^2+v^2+w^2}}.$$

And the rate-of-climb constraint is:

$$\sin\gamma = a\sin\theta - b\cos\theta. \tag{25}$$

where

$$a = \cos\alpha\cos\beta, \quad b = \sin\phi\sin\beta + \cos\phi\sin\alpha\cos\beta.$$

3.2.2 Dynamic filter

To obtain the pseudo-attitude closer to the true aircraft attitude, an extra filter is backed with the attitude synthesizer as shown in Figure 5. This filter, employing the aircraft dynamics, is appropriately named a "dynamic filter". The dynamic filter is implemented as an extended Kalman filter to manage the system's nonlinear equations.

The dynamic-filter equations in continuous domain are expressed as follows:

$$\text{Process model}: \quad \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) + \mathbf{w} \tag{26}$$

$$\text{Measurement model}: \quad \mathbf{z} = \mathbf{h}(\mathbf{x}, \mathbf{u}, t) + \mathbf{v}, \tag{27}$$

where:

$\mathbf{x}$, state vector, $\mathbf{x} = \begin{bmatrix} u & v & w & p & q & r & \psi & \theta & \phi \end{bmatrix}^{\mathrm{T}}$;

$\mathbf{u}$, control input vector, $\mathbf{u} = \begin{bmatrix} \delta e & \delta t & \delta a & \delta r \end{bmatrix}^{\mathrm{T}}$;

$\mathbf{z}$, measurement vector, $\mathbf{z} = \begin{bmatrix} u_z & p_z & q_z & r_z & \psi_z & \gamma_z & \phi_z \end{bmatrix}^{\mathrm{T}}$;

$\mathbf{w}$, processing noise vector; $\mathbf{v}$, measurement noise vector; $\mathbf{f}(\mathbf{x},\mathbf{u},t)$, function-composing equation of motion for an aircraft;

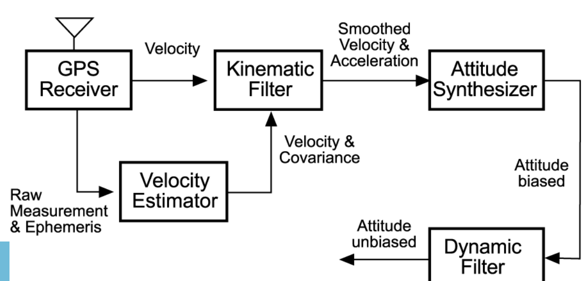
$$\mathbf{h}(\mathbf{x}, \mathbf{u}, t) = \begin{bmatrix} u & p & q & r & \psi & \gamma & \phi \end{bmatrix}^{\mathrm{T}};$$

$\gamma = \sin^{-1}(a\sin\theta - b\cos\theta)$, from equation (25) $(\ )_z$, means the measured value. Each of them is explained in following paragraphs.

The process model of equation (26) can be composed from equations (21), (22), and (23). (X, Y, Z) and (L, M, N) are determinable from the stability derivatives of the target aircraft. It should be noted that the measurement vector includes not the pitch angle but the flight path angle. However, the rate-of-climb constraint is also included in the measurement model of equation (27), which contributes to estimate the pseudo-pitch closer to the true pitch.

The measurement vector of the dynamic filter can be developed using the outputs of the attitude synthesizer as follows:

Figure 5 Block diagram with dynamic filter



- u_z : $u_z = V_{\mathrm{T}}$ can be assumed, because v and w are much smaller than u.
- p_z: calculated with equation $p = -\dot{\psi}\sin\theta + \dot{\phi}$, which is derived from kinematic equation (23). Also, $\dot{\psi}$ and $\dot{\phi}$ are substituted by real-time numerical differentiation of the attitude synthesizer's output $\tilde{\psi}$, $\tilde{\phi}$.
- ψ_z, γ_z, ϕ_z: attitude synthesizer output $\tilde{\psi}$, $\tilde{\theta}$, $\tilde{\phi}$, where $\tilde{\theta}$ still equals the flight path angle γ.
- q_z, r_z: attitude synthesizer output $\tilde{q}$, $\tilde{r}$.

States, including the unbiased single-antenna attitude, are estimated through the propagation and innovation processes of the extended Kalman filter.

There may be serious latency caused by the cascading of two Kalman filter. The kinematic filter of the existing method has to estimate the acceleration and remove the measurement noise. However, in the enhanced method, the kinematic filter can focus on only the acceleration estimation, because the dynamic filter can remove the measurement noise more appropriately. Hence, the kinematic filter is required to be tuned not to introduce the latency or replaced with simple differentiator to obtain the acceleration.

4. Simulation result

Several simulations were implemented to verify the performance of our proposed method. To show only the enhancement of the proposed method distinctly, we did not consider the some real environment: wind disturbance, uncertainty of aircraft model, etc. A summary of the simulation procedure is described below:

- Generate true states of aircraft through non-linear simulation using the Navion model.
- Exclude the wind disturbances, which the single-antenna attitude method cannot intrinsically estimate.
- Generate the satellite constellation information that comprises the positions and velocities of 24 GPS satellites arranged in 6 orbital planes with four satellites per plane.
- Prepare the 10 Hz Doppler measurements of the aircraft GPS receiver using the aircraft true state, satellite constellations, and measurement noise generated with the modelled function of the elevation angle of satellite.
- Neglect the GPS error sources degrading signal-in-space. It is reasonable to assume that the main error sources, the ionospheric and tropospheric delays, are constant and do not affect the velocity estimation in most of the earth's surface.

4.1 Satellite constellation

Satellite constellation information contributes to optimization of the kinematic filter, particularly by estimating more accurately the target's velocity and by providing its covariance value. Therefore, the simulation focuses on improving the estimated velocity when the satellite constellation fluctuates. A steady turning flight is simulated. The roll angle is set at about -50° to obtain a strong appearance.

The left-side graph in Figure 6 shows the horizontal trajectory of aircraft. The right-side graph of Figure 6 shows the number of visible satellites varying incessantly from four to seven with attitude. The covariance of estimation error will do likewise, except that the estimation error gets larger exponentially as the number of visible satellites grows smaller.

Figure 7 shows the profiles of velocity errors after the kinematic filters in the NED frame. The "On and Off" in the legend, corresponds to whether the constellation information

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Figure 6 Variation of satellite constellation in steady turning flight

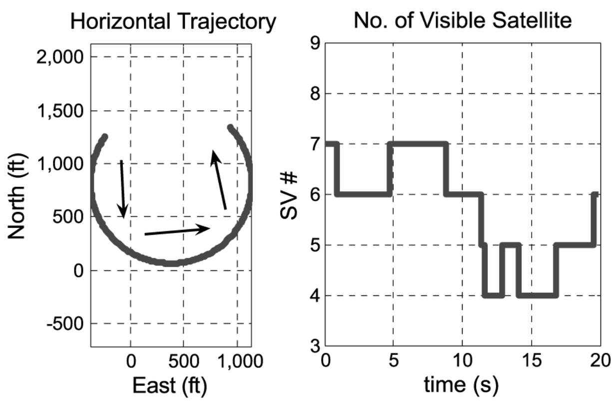
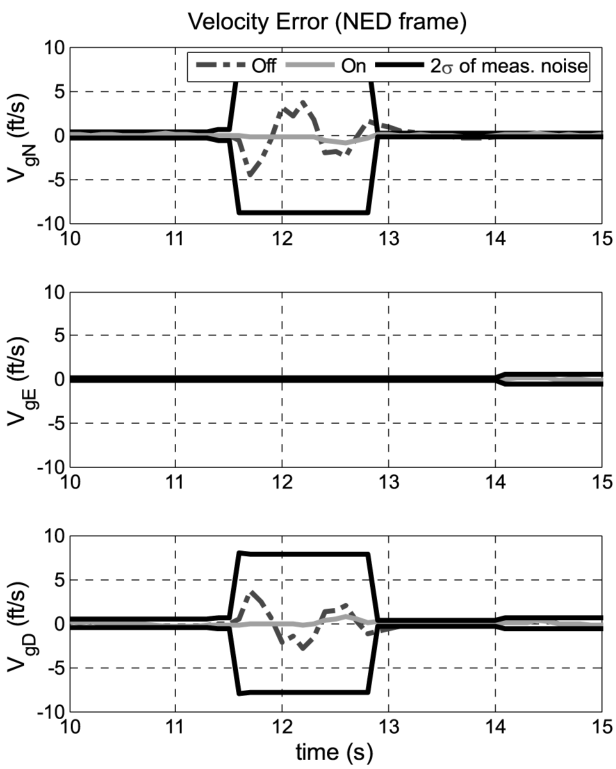


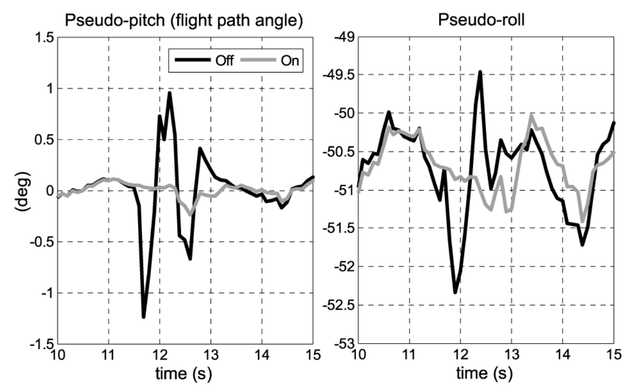
Figure 7 Velocity estimation errors in steady turning flight



is applied or not. The velocity covariance values (2σ) in each direction are depicted by dotted line. For 11 ~ 13 seconds of simulation time, only four satellites are visible and velocity covariance in north and down direction increases suddenly. Solid line denotes the velocity filtered with varying covariance determined by the satellite constellation, and shows a steady motion insensitive to even unfavorable constellations. The velocity filtered with the fixed covariance of dash – dot line is as accurate as solid line when the number of visible satellites is adequate. However, sudden blockages of satellites make the velocity errors deviate. The similar phenomena are expected to appear in the accelerations in more corrupt manner.

The estimation error of the velocity also affects the attitude. The pseudo-pitch and roll angles of two sets of velocities are shown in Figure 8. For the same period, the estimation errors in attitudes move similarly with the velocity estimation errors.

Figure 8 Pseudo-attitude in steady turning flight



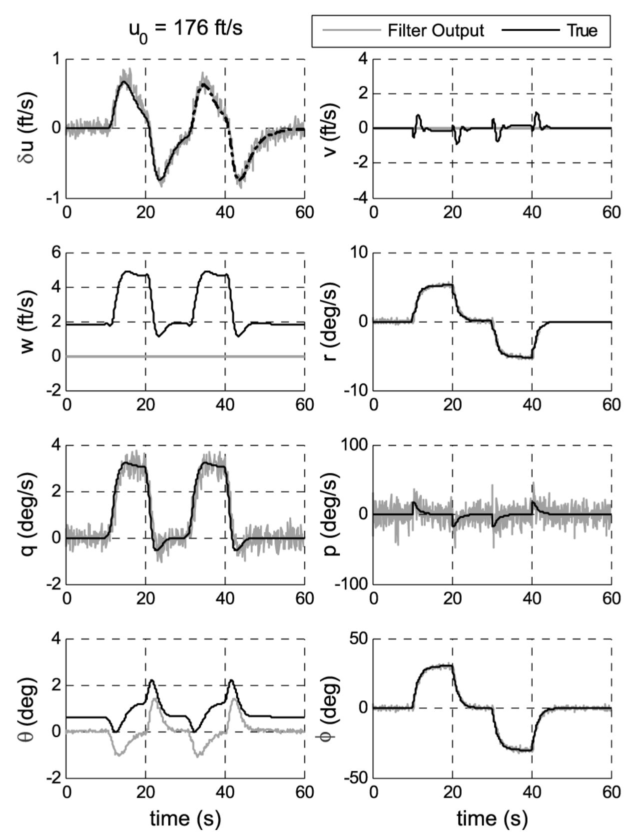
4.2 Aircraft dynamics

The biases of the single-antenna attitude are changed with the lateral motion of aircraft. Hence, a flight scenario is simulated as a series of flights that are at first cruise, then bank of ±30° and finally cruise again. For the performance comparison, three sets of states of aircraft are obtained through:

- existing method;
- proposed method without control information; and
- proposed method with control information.

Figure 9 shows a set of states estimated with the existing method consisting of kinematic filter and attitude synthesizer. The existing method assumes that the velocity axis is aligned with the body axis of aircraft. Therefore, velocity component

Figure 9 Estimated states of aircraft with existing method



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u is equal to V_T, and other components v, w are always constrained to zero. The bias of pseudo-roll is relatively small. The profile of estimated roll in Figure 9 follows well with the profile of true roll to some extent. However, pseudo-pitch from the existing method or flight path angle differs from the true pitch θ and is biased by angle of attack in cruise motion. Figure 10 is an enlarged figure of the pitch. The dotted line, which has been shifted by the trim condition θ_0, still has biases caused by lateral motion (bank).

Figure 11 shows a set of states estimated with the proposed method that employs the aircraft dynamics and rate-of-climb constraint, without control information. The estimations are much closer to the true states than those of the existing method. Especially, the pitch bias is removed, not only in the longitudinal motion but also in the lateral motion, which means that it is possible to estimate angle of attack α and sideslip angle β, from the velocity components w and v. They can be understood as the values determined analytically using any equations of motion that relate the w and v to other directly measured states such as u, p, q, and r.

Figure 12 demonstrates admirably the merits of the dynamic filter. Had the control information been available to the existing method, structural change would have been required before the system could operate. The proposed model would perform brilliantly without any modifications to its structure. In comparison to case □ the proposed method decreases the noise level outstandingly and provides estimations that follow the true states, especially in the transient part.

Figure 10 Bias of estimated pitch angle with existing method

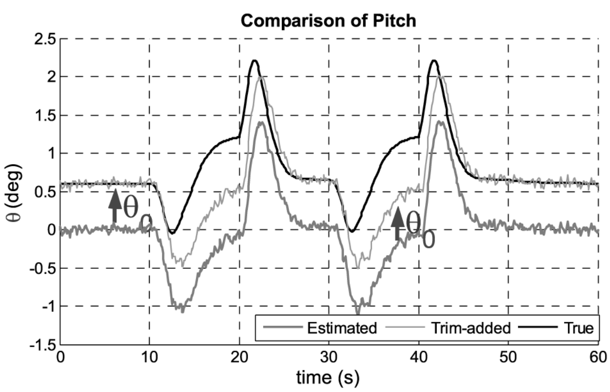
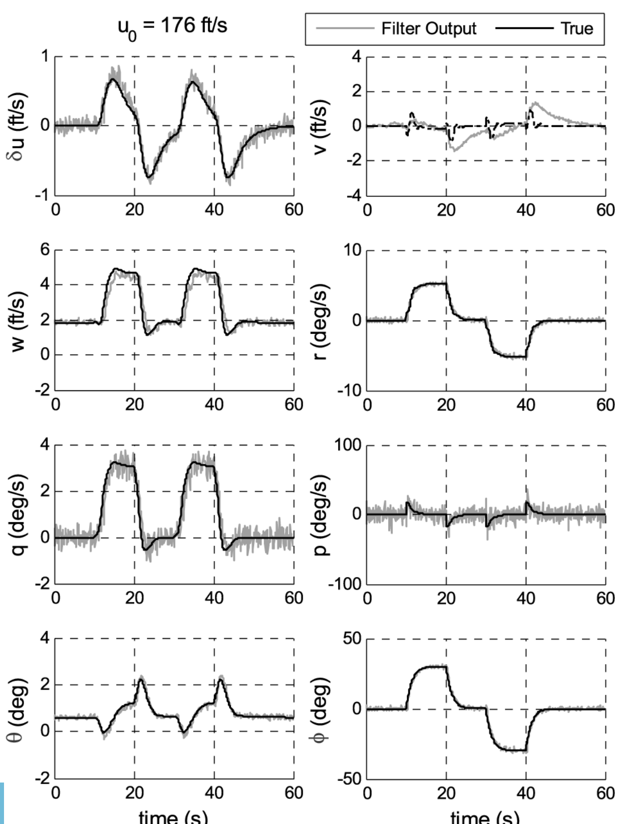


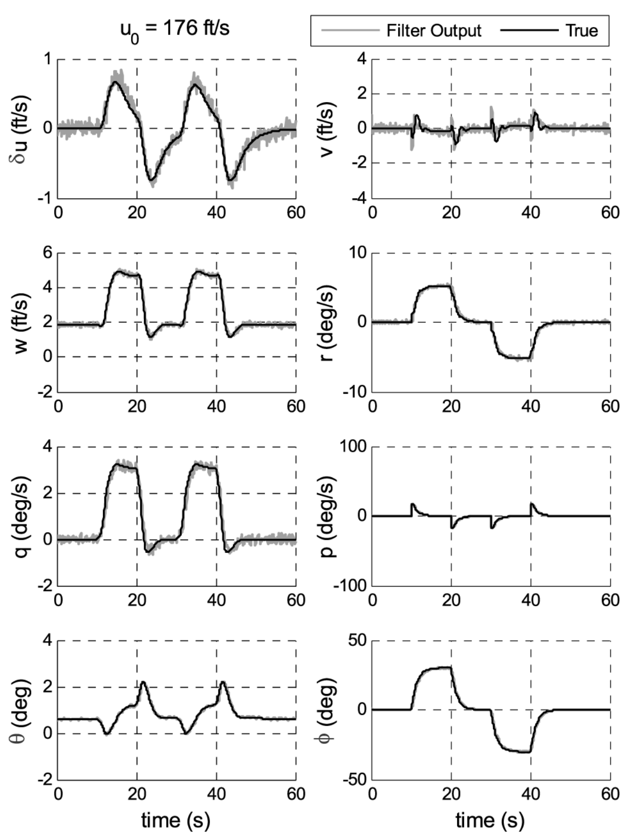
Figure 11 Estimated states of aircraft with proposed method (no control info.)



5. Discussion

Single-antenna attitude has limitations as a main attitude system from the viewpoint of safety. As noted by Kato and Sugiura (1985), it is impossible for all of the system's states to be estimated from the ground velocity or trajectory of an aircraft. However, the attitude determination system can play a useful role as a backup or sensor-failure detection system. On this basis, the performance of our proposed method is discussed.

Figure 12 Estimated states of aircraft with proposed method (control info. added)



5.1 Efficiently combined with other sensors?

In an emergency, the backup navigation system has to cooperate with other avionics after excluding the failed subsystems. When the main attitude system fails partially, there may be several sensors that can still provide normal attitude information. The existing attitude method cannot combine the information, not to mention the control input, with pseudo-attitude because of its own limited structure. However, the proposed model, with its dynamic filter, supplies an appropriate platform on which the backup attitude system can draw synergistic effects with the main attitude sensors including the single-antenna attitude. The only modification required is to add or substitute more accurate measurements obtained from the main non-failed sensors.

5.2 Why not decoupled linear equation of motion?

Coupled non-linear equations of motion make system more complicated, which may degrade system reliability. However, the proposed enhancement can be accomplished by using only the coupled non-linear equation of motion. To remove the bias in pseudo-attitude, the rate-of-climb constraint uses the longitudinal and lateral states of aircraft simultaneously. If the dynamic filter is implemented with decoupled linear equation, the constraint is forced to have a trick that uses the states of the previous epoch, which may increase the latency of estimation. Moreover, the longitudinal and lateral dynamic filters of which the process models are composed exclusively becomes inaccurate. Because the extended Kalman filter is already used widely in the aerospace industry, non-linearity does not have serious impact on system reliability.

6. Conclusion

An enhanced method has been proposed to estimate the single-antenna attitude closer to the true attitude and to combine other available information efficiently. The proposed method uses satellite constellation information to optimize the velocity estimation, and aircraft dynamics to obtain unbiased estimations of single-antenna attitude.

We verified the performance of the proposed method by simulating the single-antenna attitude system installed in the Navion aircraft and processing the GPS raw measurements obtained from visible satellites.

The error covariance of velocity estimated from the satellite constellation contributes to the optimization of velocity estimation. The dynamic filter estimates the unbiased pseudo-attitude with the equations of motion for aircraft and rate-of-climb constraint. The structure of the proposed method can easily be integrated with main sensors of aircraft.

If the single-antenna attitude model were commissioned as a backup to the main attitude system, it would be necessary to analyze this approach more practically in the areas of sensor integration, stability against wind disturbance and aircraft model uncertainty, etc.

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Further reading

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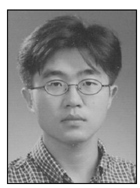

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